



# Quantum annealing and its variants: Application to quadratic unconstrained binary optimization

Vrinda Mehta

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