



# Real-time simulations of transmon systems with time-dependent Hamiltonian models

Hannes Alfred Lagemann

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