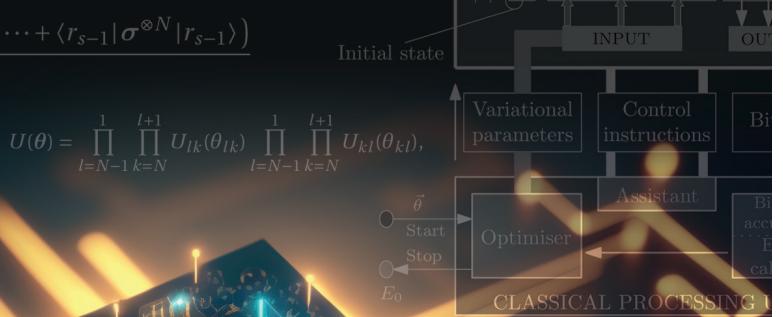
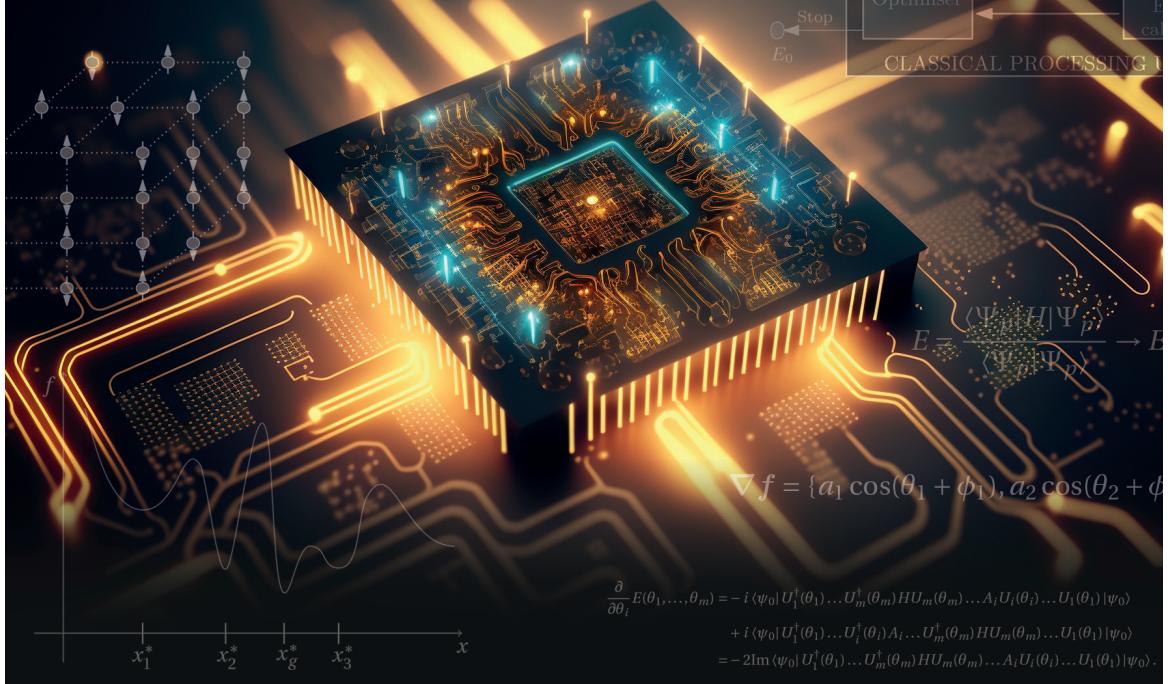


$$\begin{aligned}
&= \frac{c_j (\langle r_0 | \sigma^{\otimes N} | r_0 \rangle + \langle r_1 | \sigma^{\otimes N} | r_1 \rangle + \dots + \langle r_{s-1} | \sigma^{\otimes N} | r_{s-1} \rangle)}{s} \\
&= \frac{c_j (\mathcal{P}_0 + \mathcal{P}_1 + \dots + \mathcal{P}_{s-1})}{s} \\
&= \frac{c_j (\sum_{i=0}^{s'-1} \mathcal{P}_i \mathcal{G}(r_i, R))}{s},
\end{aligned}$$



$$U(\theta) = \prod_{l=N-1}^1 \prod_{k=N}^{l+1} U_{lk}(\theta_{lk}) \prod_{l=N-1}^1 \prod_{k=N}^{l+1} U_{kl}(\theta_{kl}),$$



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