



A Unified Framework for Functional Renormalisation Group Calculations and its Application to Three Dimensional Hubbard Models

Jannis Ehrlich

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