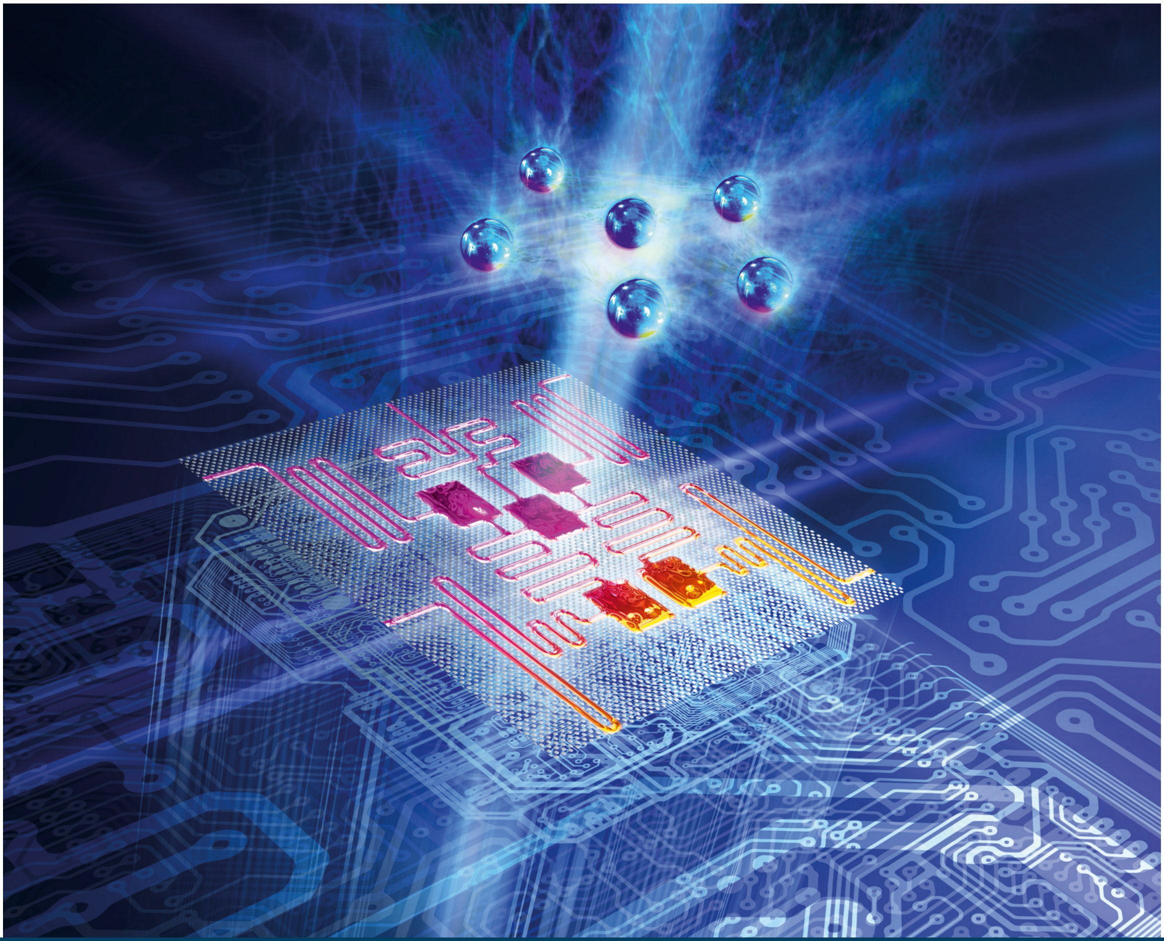

Supercomputer simulations of transmon quantum computers

Dennis Willsch

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