



# **The method of fundamental solutions for computing interior transmission eigenvalues**

Lukas Julian Pieronek

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