



# Inspection Games over Time

## Fundamental Models and Approaches

Rudolf Avenhaus, Thomas Krieger

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