

## Model-based reconstruction of magnetisation distributions in nanostructures from electron optical phase images

Jan Caron

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