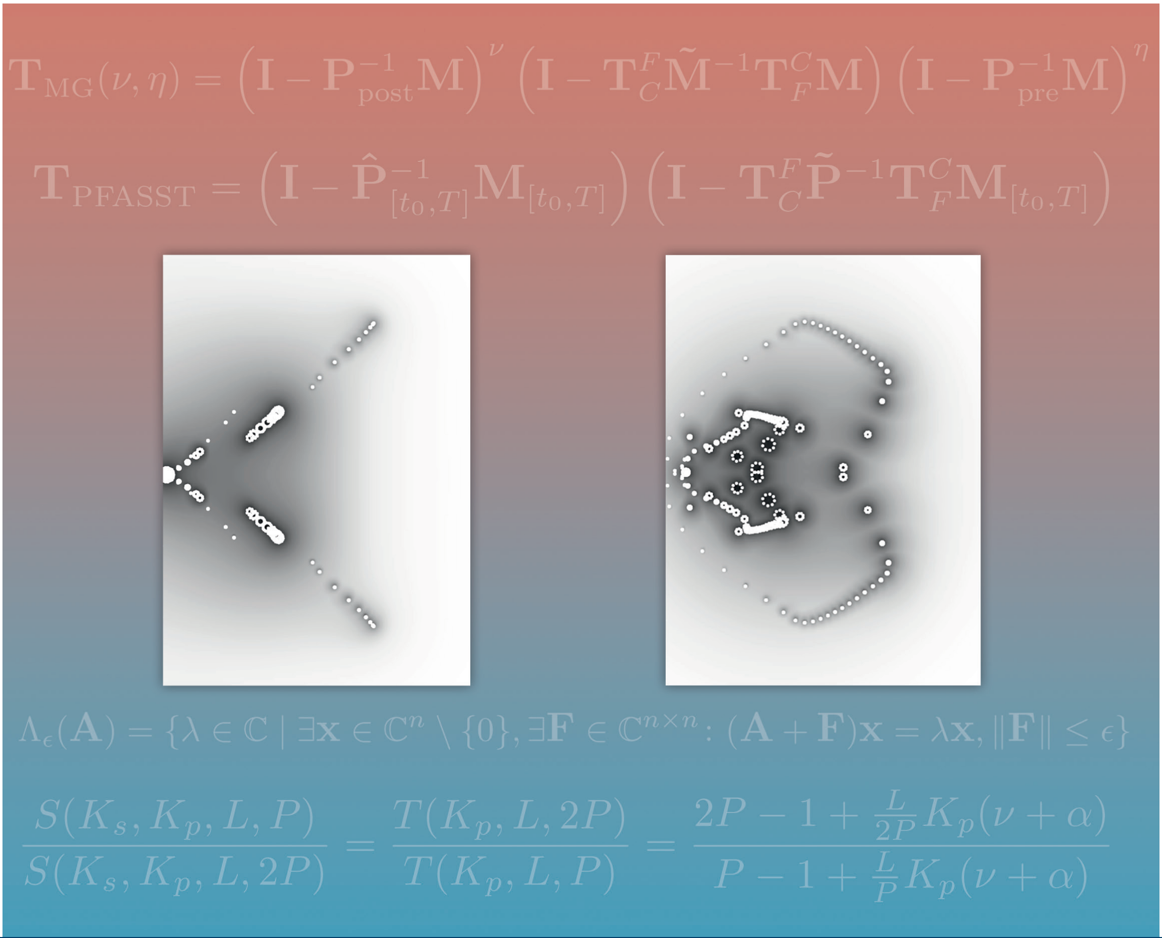

A multigrid perspective on the parallel full approximation scheme in space and time

Dieter Moser

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